Example Proofs Involving Divisibility (NTS = "Need to Show")

To Prove: For all integers m and n, if $6 \mid m$ and $4 \mid n$, then $2 \mid (5m - 7n)$.

Proof: Let m and n be integers.
Suppose that $6 \mid m$ and $4 \mid n$.
Then, $m = 6k$ and $n = 4p$ for some integers k and p , by definition of "divides" .
$\therefore$ $5m - 7n = 5(6k) - 7(4p)$, by substitution,
$= 30k - 28p$ by R. O. A. .

[Need to show: $5m - 7n = 2t$ for some integer t .]

From above, $5m - 7n = 30k - 28p$
$= 2(15k - 14p)$ by R. O. A. .

Let $t = (15k - 14p)$, which is an integer.
$\therefore$ $5m - 7n = 2t$, by substitution , and t is an integer..
$\therefore$ $2 \mid (5m - 7n)$ by definition of "divides".
$\therefore$ For all integers m and n, if $6 \mid m$ and $4 \mid n$, then $2 \mid (5m - 7n)$, by Direct Proof.
Q E D

Theorem 4.3.3 (Page 137): "Divisibility is Transitive;" that is, for all integers a , b, and c , if $a \mid b$ and $b \mid c$, then $a \mid c$.

Proof: Let a, b, and c be any integers.

Suppose $a \mid b$ and $b \mid c$. [NTS: $a \mid c$. NTS $c = at$ for some integer t .]

By definition of "divides," there exist integers k and p such that $b = ak$ and $c = bp$.

$\therefore$ $c = (ak)p$ by substitution of b by (ak) in the equation $c = bp$,
$= a(kp)$ by R. O. A. .

Let $t = (kp)$, which is an integer, because products of integers are integers.
$\therefore$ $c = at$ by substitution , and t is an integer.

$\therefore$ $a \mid c$, by definition of "divides".

$\therefore$ For all integers a , b, and c , if $a \mid b$ and $b \mid c$, then $a \mid c$, by Direct Proof.

Q E D

To Prove: For all integers a, b, and c , if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

Proof: Let a, b, and c be any integers.
Suppose that $a \mid b$ and $a \mid c$.

Then, $b = a k$ and $c = a p$ for some integers k and p by definition of "divides" .

$\therefore$ $(b + c) = a k + a p$ by substitution,

$= a(k + p)$ by R. O. A.

$= a t$, where t is the integer such that $t = (k + p)$.

$\therefore$ $(b + c) = a t$, and t is an integer .

$\therefore$ $a \mid (b + c)$ by definition of "divides."

$\therefore$ For all integers a, b, and c , if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$, by Direct Proof.

Q E D